

Linear Algebra

* REF $\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$ leading non-zero has below whole column zero.

* RREF $\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow$ leading non-zero is 1 if has below and above entries zero.

$$* A^{-1} \Rightarrow (cA)^{-1} = \frac{1}{c} A^{-1}, (A^k)^{-1} = (A^{-1})^k, (A^T)^{-1} = (A^{-1})^T$$

$$\rightarrow |A| \Rightarrow \det(AB) = \det A \cdot \det B$$

$$\Rightarrow \det(kA) = k^n \det(A)$$

$$\Rightarrow \det(A^{-1}) = \frac{1}{\det(A)}$$

$$\det(B) = -\det(A) \rightarrow \text{B is matrix obtained by interchanging over two rows.}$$

→ Vector spaces :-

(i) $\rightarrow \alpha X + \beta Y \in S \rightarrow$ closure under addition

(ii) $\rightarrow \alpha X \in S \rightarrow$ closure under multiplication

"These two properties are used to prove not a subspace."

(iii) $\rightarrow$ Span of a vector space is vectors of that vector space which can be used to generate whole vector space.

$$\text{Ex: } \{(0,1), (1,0)\} \rightarrow \text{Span of } \mathbb{R}^2$$

$$\text{Span}(\{(0,1), (1,0)\}) = \mathbb{R}^2$$

(iv) To check whether given vectors are L.D or not put them in matrix where each row is vector if any row gets $(0,0,0,\dots)$ it is L.D.

(v) A set which is (i) L.I & (ii) s.t's shown is equal to a vector space, then it is basis set of that vector space.

(vi) Dimension :- No. of elements in the basis set.

(vii) Every Linearly Independent set in V can be extended to form the basis set.

(viii) Any two basis set of V will have same elements

(ix) $\dim(V) = n$, then any set containing $n+1$ vectors will be L.D.

* Extending a given L.I $\rightarrow$ basis set.

$$\{(1, 1, 0, 0), (0, 0, 1, 1)\} = \text{Basis??}$$

$\Rightarrow$ let En Basis be $\{v_1, v_2, e_1, e_2, e_3, e_4\} \Rightarrow$ L.D

Only $\leftarrow$ use these two rows in operations

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \text{basis} = \{(1, 1, 0, 0), (0, 0, 1, 1), (0, 1, 0, 0), (0, 0, 1, 0)\}$$

* Extracting basis from L.D vectors.

S-1) Put them in matrix where each row is vector.

S-2) Create Row echelon form, remove all rows which are zero at the end.

* Transformations :-

$$T(\alpha x + \beta y) = \alpha T(x) + \beta T(y)$$

$$T(\alpha x) = \alpha T(x)$$

$$N(T) = \text{Null space of } T = \{x \in V : Tx = 0\}$$

$$R(T) = \text{Range of } T = \{Tx : x \in V\}$$

Range Nullity theorem $\Rightarrow$

$$\dim(N(T)) + \dim(R(T)) = \dim(V)$$

then the transformation is valid.

Given transformation on basis set to find the transformation in general.

$$\text{Ex: } T(2,3) = (4,5), \quad T(1,0) = (0,0)$$

$$(x,y) = \alpha(2,3) + \beta(1,0)$$

$$\Rightarrow \therefore \alpha = \frac{y}{3}, \quad \beta = x - \frac{2y}{3}$$

$$T(x,y) = \alpha T(2,3) + \beta T(1,0)$$

$$\Rightarrow T(x,y) = \frac{y}{3} T(2,3) + \beta(0,0)$$

$$\Rightarrow T(x,y) = \left(\frac{4y}{3}, \frac{5y}{3}\right) \quad \underline{\underline{\text{Ans}}}$$

Eigen values :-

Sum of Eigen values = Trace of Matrix.

Product of Eigen values = Det of Matrix.

A & A^T have same eigen values.

$\rightarrow \lambda_1, \lambda_2, \lambda_3 \dots$ are Eigen values of A
 then $A^m \rightarrow \lambda_1^m, \lambda_2^m, \lambda_3^m \dots$

Same for $kA \rightarrow k\lambda_1, k\lambda_2 \dots$

$\rightarrow$ Eigen values for $A^{-1} = \frac{1}{\lambda_1}, \frac{1}{\lambda_2} \dots$

$\rightarrow$ If Eigen value = 0 then $\det(A) = 0$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix} \rightarrow \text{Eigen values}$$

A.M $\Rightarrow$ Number of times a Eigen value is repeating

G.M $\Rightarrow$ Eigen vector for that eigen value has how many zero row in row echelon form.

If $AM = GM$ then it is diagonalizable.

* Orthogonal vectors:-

$$\{n_1, n_2, n_3\} \quad ||v_1|| = \sqrt{a^2 + b^2 \dots}$$

$$\Rightarrow \vec{v}_1 = \vec{n}_1$$

$$\vec{v}_2 = \vec{n}_2 - \frac{(\vec{v}_1 \cdot \vec{n}_2) \vec{v}_1}{||\vec{v}_1||^2}$$

$$\vec{v}_3 = \vec{n}_3 - \frac{(\vec{v}_1 \cdot \vec{n}_3) \vec{v}_1}{||\vec{v}_1||^2} - \frac{(\vec{v}_2 \cdot \vec{n}_3) \vec{v}_2}{||\vec{v}_2||^2}$$