Digital Design

Binary Arithmetic & Complements

Introduction

- A Computer understands only Binary
- The Arithmetic and Logical operation is done also in Binary

3 + 5

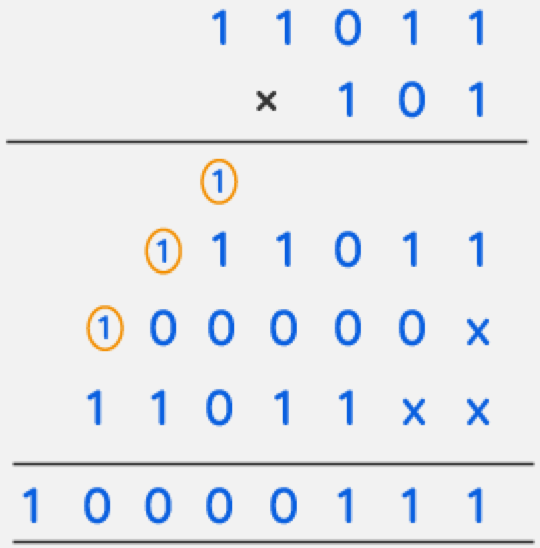
= 8

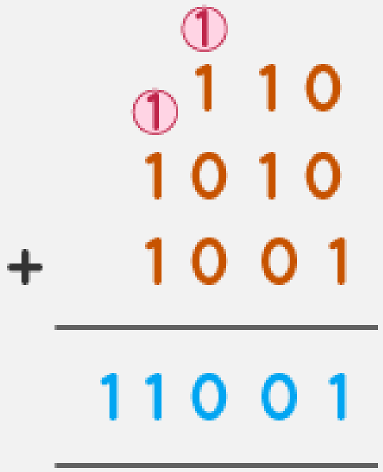
0011 + 0101

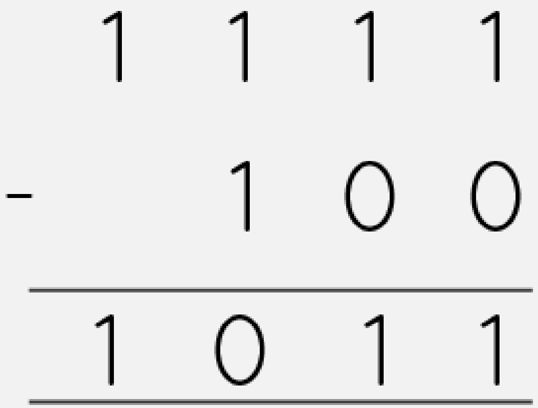
= 1000

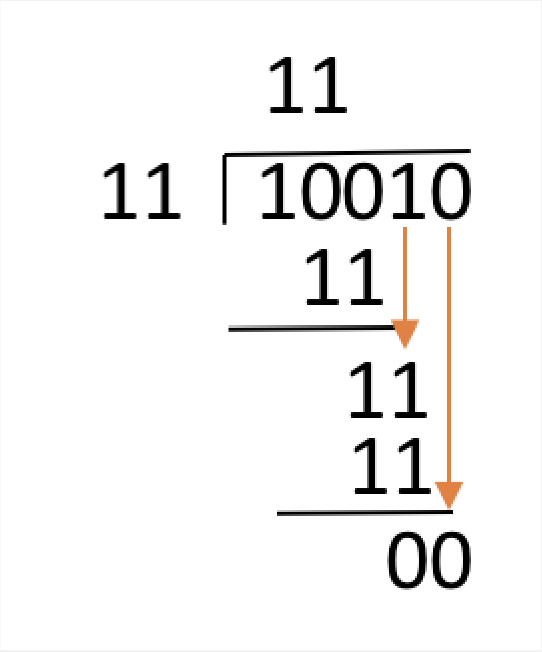
Binary Arithmetic

- Addition
- Subtraction
- Multiplication
- Division









Binary Addition

- Adding bits:
  - 0 + 0 =   0
  - 0 + 1 =   1
  - 1 + 0 =   1
  - 1 + 1 = (1) 0
- Adding integers:

				1	1	0			
	0	0	0	0	1	1	1	$_{two}$ =	7_{ten}
+	0	0	0	0	1	1	0	$_{two}$ =	6_{ten}
=	0	0	0	1	(1)1	(1)0	(0)1	$_{two}$ =	13_{ten}

Binary Subtraction

- Bitwise:
  - 0 - 0 = 0
  - 0 - 1 = (1)1
  - 1 - 0 = 1
  - 1 - 1 = 0
- Direct Subtraction:

$$
\begin{array}{rccccccl}
 & 0 & 0 & 0 \ldots\ldots 0 & 1 & 1 & 1_{two} & = 7_{ten} \\
- & 0 & 0 & 0 \ldots\ldots 0 & 1 & 1 & 0_{two} & = 6_{ten} \\
\hline
= & 0 & 0 & 0 \ldots\ldots 0 & 0 & 0 & 1_{two} & = 1_{ten}
\end{array}
$$

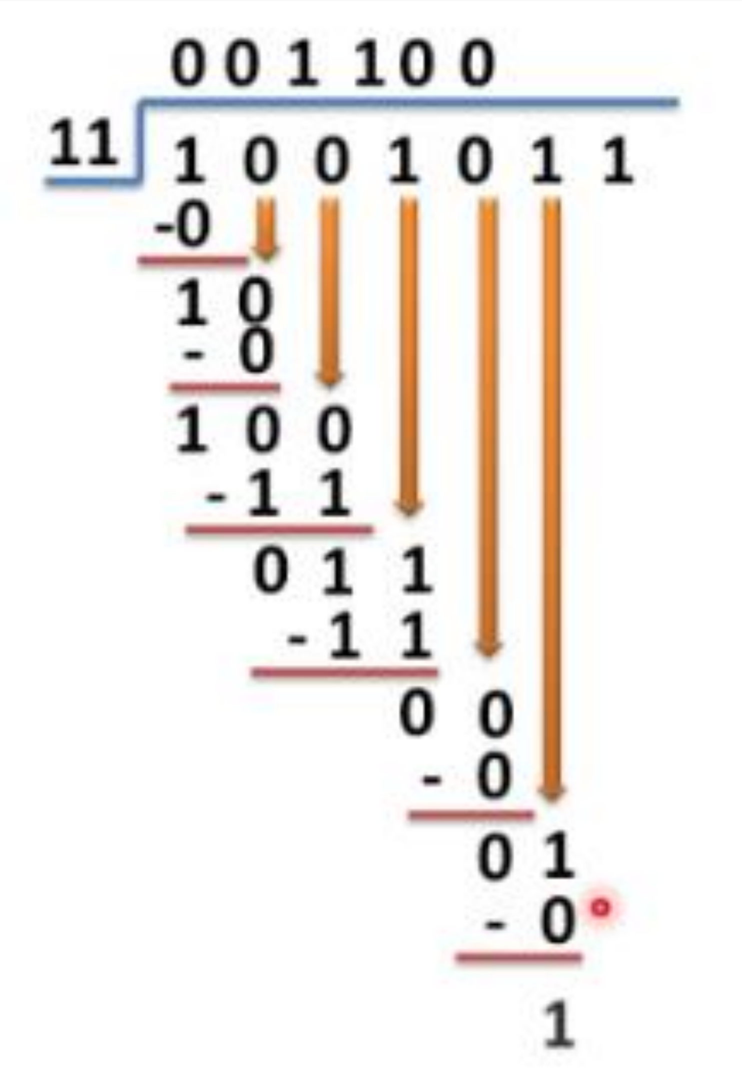
Binary Multiplication

- Bitwise:
  - 0 * 0 = 0
  - 0 * 1 = 0
  - 1 * 0 = 0
  - 1 * 1 = 1
- Direct Multiplication:

1000_{two}	$= 8_{ten}$	multiplicand
1001_{two}	$= 9_{ten}$	multiplier
1000		partial products
0000		
0000		
1000		
1001000_{two}	$= 72_{ten}$	

Binary Division

- Bitwise:
  - 0 / 0 = not defined
  - 0 / 1 = 0
  - 1 / 0 = not defined
  - 1 / 1 = 1
- Direct Division:



Exercise

- 1111 + 1010 Ans: 11001
- 1101 – 1001 Ans: 0100
- 1001 – 0111 Ans: 0010
- 1011 * 1101 Ans: 10001111
- 11001 / 101 Ans: 101

Complements

- Why Complements ….?
  - To simplify the subtraction operation and for logical manipulation
  - Computer does not directly subtract, it adds…

- Two Types of Complements
  - (r-1)’s complement
  - r’s complement

r is the radix of a given number

(r-1)'s Complements

- Given a number 'N' in base 'r' having 'n' digits
- The r's complement is given by:

$$(r^n - 1) - N$$

- Hence in computing system: there exists
  - 9's complement
  - 7's complement
  - 1's complement
  - 15's complement

(r-1)'s Complements

- Example $(r^n - 1) - N$

For $r = 10$, $N = 1234_{10}$, $n = 4$ (4 digits),we have:

$$(r^n - 1) = 10{,}000 - 1 = 9999_{10}$$

The <u>9's complement</u> of 1234_{10} is then:

$$9999_{10} - 1234_{10} = 8765_{10}$$

r's Complements

- Given a number 'N' in base 'r' having 'n' digits
- The r's complement is given by:

$$\begin{cases} r^n - N & \text{for } N \neq 0 \text{ and} \\ 0 & \text{for } N = 0 \end{cases}$$

- Hence in computing system: there exists
  - 10's complement
  - 8's complement
  - 2's complement
  - 16's complement

r's Complements

- Example $r^n - N$ for $N \neq 0$ and
  0 for $N = 0$

For $r = 10$, $N = 1234_{10}$, $n = 4$ (4 digits), we have:
$r^n = 10{,}000_{10}$
The 10's complement of 1234_{10} is then

$10{,}000_{10} - 1234_{10} = 8766_{10}$ or 8765 + 1 (9's complement plus 1)

- Important note from the equation

Note that the Radix Complement is obtained by <u>adding 1</u> to the Diminished Radix Complement.

Binary 1's Complement Example

For $r = 2$, $N = 01110011_2$, $n = 8$ (8 digits), we have:

$$(r^n - 1) = 256 - 1 = 255_{10} \text{ or } 11111111_2$$

The 1's complement of 01110011_2 is then:

$$\begin{array}{r} 11111111_2 \\ -\ 01110011_2 \\ \hline 10001100_2 \end{array}$$

NOTE: Since the 2^n-1 factor consists of all 1's and since 1 - 0= 1 and 1 - 1 = 0, forming the one's complement consists of complementing each individual bit.

Binary 2's Complement Example

For $r = 2$, $N = 01110011_2$, $n = 8$ (8 digits), we have:

$$(rn) = 256_{10} \text{ or } 100000000_2$$

The 2's complement of 01110011_2 is then:

$$
\begin{array}{r}
100000000_2 \\
-\ 01110011_2 \\
\text{----------------} \\
10001101_2
\end{array}
$$

Note that this is the 1's complement plus 1.

Shortcuts

Exercise

- Find 1's complement of: 1000, 1010, 0101
- Find 2's complement of: 1000, 1010, 0101