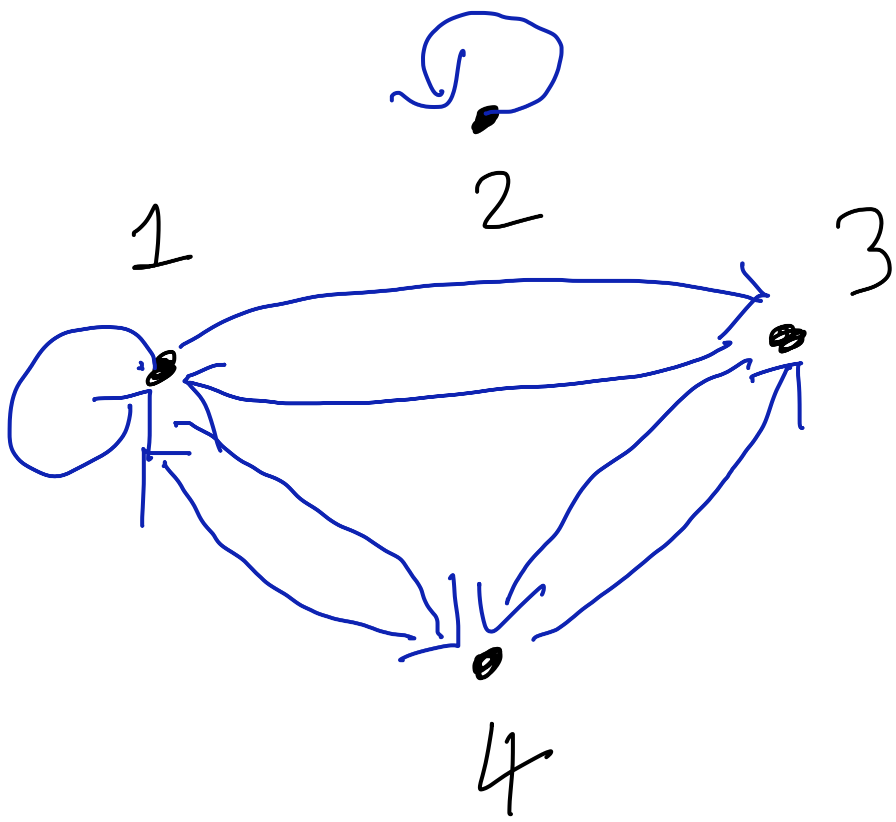
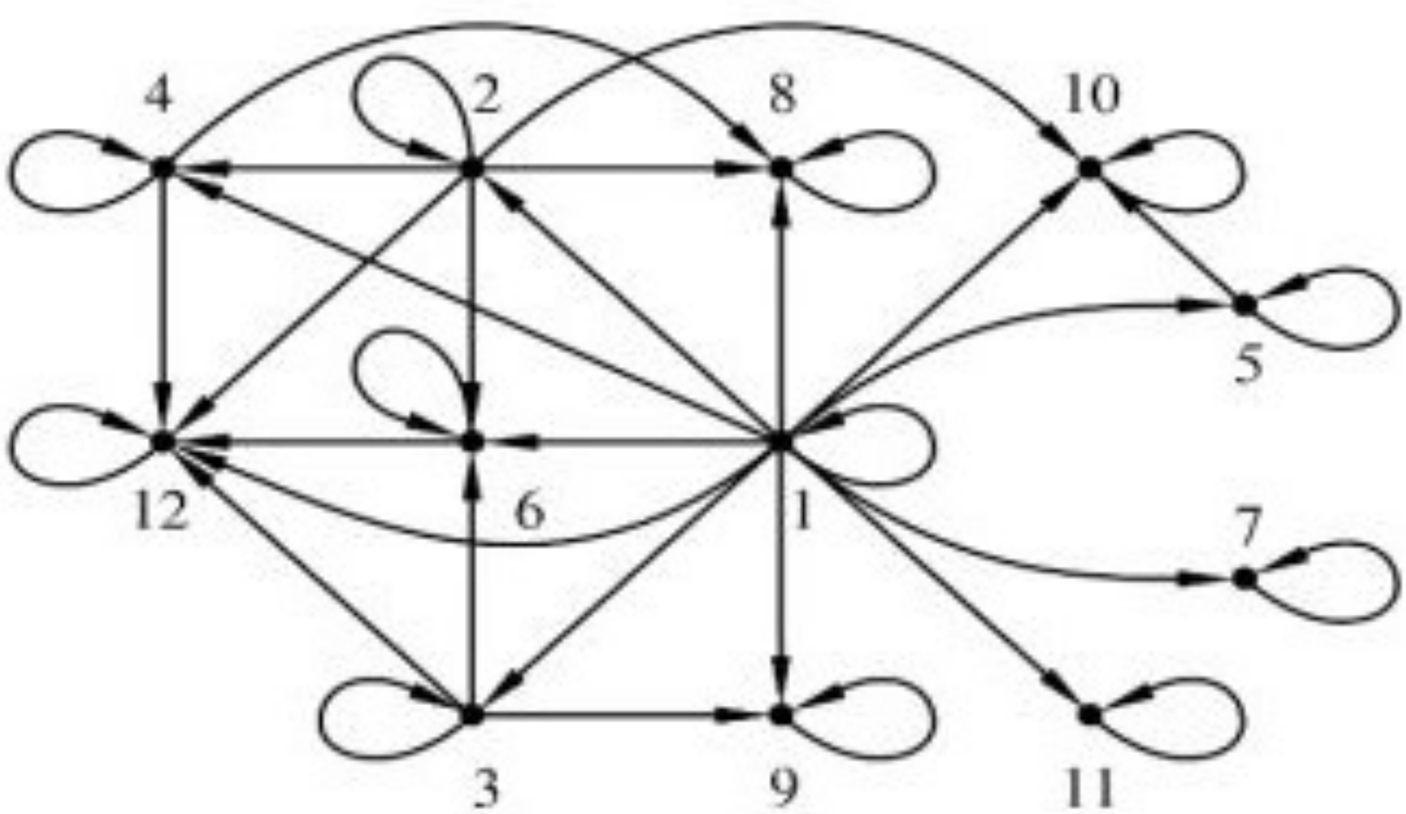
Solution Tut - 7

1)

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$



2)



3) $M_{R_1} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ & $M_{R_2} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

a) As all diagonal entries of M_{R_1} are 1, R_1 is reflexive.

→ R_1 is antisymmetric

as $m_{ij} = 1 \Rightarrow m_{ji} = 0$; $i \neq j$

→ R_1 is transitive

as $M_{R_1} + M_{R_1}^2 + M_{R_1}^3 = M_{R_1}$

b) In M_{R_2} all diagonal entries are 0, therefore R_1 is irreflexive.

→ R_2 is symmetric

as $m_{ij} = m_{ji}$ $\forall$ i, j

→ R_2 is not transitive

as $(1, 2)$ & $(2, 1) \in R_2$ but $(1,1) \notin R_2$

c) $M_{R_1^c} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$$M_{R_1 \cup R_2} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$M_{R_1 \cap R_2} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Here $R_1 = \{(1,1), (1,2), (1,3), (2,2), (3,2), (3,3)\}$

& $R_2 = \{(1,2), (2,1), (2,3), (3,2)\}$

$\therefore R_1 \circ R_2 = \{(1,2), (2,1), (2,2), (2,3), (3,2)\}$

$$\therefore M_{R_1 \circ R_2} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

<u>or</u>

$$M_{R_1 \circ R_2} = M_{R_2} \cdot M_{R_1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} =$$

4) $S = \{1, 2, 3, 4, 5\}$

$R = \{(1, 2), (2, 2), (2, 3), (5, 4)\}$

Reflexive Closure:

Here $I_S = \{(1,1), (2,2), (3,3), (4,4), (5,5)\}$

$\therefore$ Reflexive closure of $R = R^* = R \cup I_S$

$\therefore R^* = \{(1,1), (1,2), (2,2), (2,3), (3,3), (4,4), (5,4), (5,5)\}$

Symmetric Closure:-

$R^{-1} = \{(2,1), (2,2), (3,2), (4,5)\}$

$\therefore$ Symmetric Closure $= R^S = R \cup R^{-1}$

$R^S = \{(1,2), (2,1), (2,2), (2,3), (3,2), (4,5), (5,4)\}$

Transitive Closure:-

$R = \{(1,2), (2,2), (2,3), (5,4)\}$

$R^2 = R \circ R = \{(1,2), (1,3), (2,2), (2,3)\}$

$R^3 = R \circ R^2 = \{(1,2), (1,3), (2,2), (2,3)\}$

$R^5 = R^4 = \{(1,2), (1,3), (2,2), (2,3)\}$

$\therefore R^T = R \cup R^2 \cup R^3 \cup R^4 \cup R^5$

$= \{(1,2), (1,3), (2,2), (2,3), (5,4)\}$

5) $f(x) = 3x^3 + 2x^2 - 5x + 2$

$g(x) = -x^3 - x^2 + 5$

Now

$(f+g)(x) = f(x) + g(x)$

$= 3x^3 + 2x^2 - 5x + 2 - x^3 - x^2 + 5$

$= 2x^3 + x^2 - 5x + 7$

$(f-g)(x) = f(x) - g(x)$

$= 3x^3 + 2x^2 - 5x + 2 + x^3 + x^2 - 5$

$= 4x^3 + 3x^2 - 5x - 3$.

$(f \cdot g)(x) = f(x) \cdot g(x)$

$= (3x^3 + 2x^2 - 5x + 2)(-x^3 - x^2 + 5)$

$= -3x^6 - 3x^5 + 15x^3 - 2x^5 - 2x^4 + 10x^2$
$+ 5x^4 + 5x^3 - 25x - 2x^3 - 2x^2 + 10$

$= -3x^6 - 5x^5 + 3x^4 + 18x^3 + 8x^2 - 25x + 10$

$f(g(x)) = 3(-x^3 - x^2 + 5)^3 + 2(-x^3 - x^2 + 5)^2$
$- 5(-x^3 - x^2 + 5) + 2$

6) For $f(x) = x^2 \Rightarrow f^{-1}(x) = x^{1/2}$.

Now Let $f(x) = f(y)$

$\Rightarrow x^2 = y^2$

$\Rightarrow x^2 - y^2 = 0$

$\Rightarrow (x-y)(x+y) = 0$

$\Rightarrow x = y$ or $x = -y$

a) Here $f : \mathbb{R} \to \mathbb{R}$.

$\therefore f(x) = f(y) \not\Rightarrow x = y$

$\therefore f$ is not injective.

& for any -ve real number

$-x$, $(-x)^{1/2} \notin \mathbb{R}$

$\therefore$ -ve real number have no preimage under f.

$\therefore f$ is not surjective.

b) $f : \mathbb{R} \to \mathbb{R}_{\geq 0}$

$\therefore f(x) = f(y) \not\Rightarrow x = y$

$\therefore f$ is not injective.

Now, for any $x \in \mathbb{R}_{\geq 0} \Rightarrow (x)^{1/2} \in \mathbb{R}$

$\therefore f$ is surjective.

c) $f : \mathbb{R}\geq 0 \to \mathbb{R}\geq 0$

$f(x) = f(y) \Rightarrow x = y$ (as $x = -y$ is not possible)

$\therefore$ f is injective.

Now for any $x \in \mathbb{R}\geq 0 \Rightarrow (x)^{1/2} \in \mathbb{R}\geq 0$

$\therefore$ f is surjective.

Hence, f is bijective.

(7) $f : \mathbb{N} \to \mathbb{R}\setminus\{1\}$

$$f(x) = \frac{x+1}{x+2}$$

To check one-one →

let $f(a) = f(b)$

$$\frac{a+1}{a+2} = \frac{b+1}{b+2}$$

$$(a+1)(b+2) = (b+1)(a+2)$$

$$ab+b+2a+2 = ab+a+2b+2$$

$$a = b$$

$\Rightarrow$ f is one-one

To check f is onto →

let $f(x) = y$

$$\frac{x+1}{x+2} = y$$

$$x+1 = xy+2y$$

$$x(1-y) = 2y-1$$

$$x = \frac{2y-1}{1-y}$$

so, x exists for $\forall y \in \mathbb{R}\setminus\{1\}$

So f is onto.

So, f is onto

f is one-one and onto so f^{-1} exists.

To find f^{-1} →

$$\text{let } f(x) = y$$

$$\frac{x+1}{x+2} = y$$

$$x+1 = xy + 2y$$

$$x(1-y) = 2y-1$$

$$x = \frac{2y-1}{1-y}$$

$$\boxed{f^{-1}(y) = \frac{2y-1}{1-y}}$$

is inverse function of function f.