Tutorial Sheet 3 Solutions DMS

1) a) Amar is neither rich nor happy.

b) Virat is neither smart nor hard working.

c) Leela will not move to Bangalore and Delhi.

2) a) LHS $= \neg(p \vee (\neg p \wedge q))$

$\equiv \neg p \wedge (\neg(\neg p \wedge q))$ De Morgan's law

$\equiv \neg p \wedge (\neg(\neg p) \vee \neg q)$ De Morgan's law

$\equiv \neg p \wedge (p \vee \neg q)$ Double negation law

$\equiv (\neg p \wedge p) \vee (\neg p \vee \neg q)$ Distributive law

$\equiv F \vee (\neg p \vee \neg q)$ Negation law

$\equiv (\neg p \vee \neg q) \vee F$ Commutative law of disjunction

$\equiv \neg p \vee \neg q$ identity law

$=$ RHS $\square$

b) LHS $= (p \to q) \wedge (p \to r)$

$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$ (implication law)

$\equiv \neg p \vee (q \wedge r)$ (Distributive law)

$\equiv p \to (q \wedge r)$ (implication law)

$=$ RHS $\square$

c) LHS $(p \to r) \wedge (q \to r) \equiv (\neg p \vee r) \wedge (\neg q \vee r)$ (implication law)

$\equiv (\neg p \wedge \neg q) \vee r$ (Distributive law)

$\equiv \neg(p \vee q) \vee r$ (De Morgan's law)

$\equiv (p \vee q) \rightarrow r$ (implication law)

$= RHS$ $\square$

d) LHS $= (p \rightarrow q) \vee (p \rightarrow r)$

$\equiv (\neg p \vee q) \vee (\neg p \vee r)$ (implication law)

$\equiv \neg p \vee (q \vee \neg p) \vee r$ (associative law)

$\equiv \neg p \vee (\neg p \vee q) \vee r$ (commutative law)

$\equiv (\neg p \vee \neg p) \vee (q \vee r)$ (associative law)

$\equiv \neg p \vee (q \vee r)$ (Idempotent law)

$\equiv p \rightarrow (q \vee r)$ (implication law)

$= RHS$ $\square$

e) LHS $= (p \rightarrow r) \vee (q \rightarrow r)$

$\equiv (\neg p \vee r) \vee (\neg q \vee r)$ (implication law)

$\equiv \neg p \vee (r \vee \neg q) \vee r$ (associative law)

$\equiv \neg p \vee (\neg q \vee r) \vee r$ (commutative law)

$\equiv (\neg p \vee \neg q) \vee (r \vee r)$ (associative law)

$\equiv \neg (p \wedge q) \vee r$ (De Morgan's law and Idempotent law)

$\equiv (p \wedge q) \rightarrow r$ (implication law)

$= RHS$ $\square$

f)

$$\text{LHS} = \neg p \to (q \to r)$$

$$\equiv \neg(\neg p) \vee (q \to r) \quad \text{(implication law)}$$

$$\equiv p \vee (\neg q \vee r) \quad \text{(double negation law and implication law)}$$

$$\equiv (p \vee \neg q) \vee r \quad \text{(associative law)}$$

$$\equiv (\neg q \vee p) \vee r \quad \text{(commutative law)}$$

$$\equiv \neg q \vee (p \vee r) \quad \text{(associative law)}$$

$$\equiv q \to (p \vee r) \quad \text{(implication law)}$$

$$= \text{RHS} \quad \square$$

g) $\text{LHS} = p \leftrightarrow q$

$$\equiv (p \to q) \wedge (q \to p) \quad \text{(biconditional implication)}$$

$$\equiv (\neg q \to \neg p) \wedge (\neg p \to \neg q) \quad \text{(contrapositive law)}$$

$$\equiv (\neg p \to \neg q) \wedge (\neg q \to \neg p) \quad \text{(Commutative law)}$$

$$\equiv \neg p \leftrightarrow \neg q \quad \text{(biconditional law)}$$

$$= \text{RHS} \quad \square$$

h) $\text{LHS} = p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$$

$$\equiv (\neg p \wedge (\neg q \vee p)) \vee (q \wedge (\neg q \vee p))$$

(Distributive law)

$$\equiv ((\neg p \wedge \neg q) \vee (\neg p \wedge p)) \vee ((q \wedge \neg q) \vee (q \wedge p))$$

(Distributive law)

$$\equiv ((\neg p \wedge \neg q) \vee F) \vee (F \vee (q \wedge p))$$

$$((T \wedge \neg q) \vee F) \vee (F \vee (q \wedge p))$$

(Negation law)

$$\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$$

(Identity law)

$$\equiv (\neg p \wedge \neg q) \vee (p \wedge q)$$

(Commutative law)

$$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$

(commutative law)

$= RHS \quad \square$

3) a) $((p \vee q) \wedge \neg p) \rightarrow q$

$\equiv ((p \wedge \neg p) \vee (q \wedge \neg p)) \rightarrow q$ (Distributive law)

$\equiv (F \vee (q \wedge \neg p)) \rightarrow q$ (Negation law)

$\equiv (q \wedge \neg p) \rightarrow q$ (Identity law)

$\equiv \neg (q \wedge \neg p) \vee q$ (implication law)

$\equiv (\neg q \vee \neg(\neg p)) \vee q$ (De Morgan's law)

$\equiv (\neg q \vee p) \vee q$ (Double negation law)

$\equiv (p \vee \neg q) \vee q$ (Commutative law)

$\equiv p \vee (\neg q \vee q)$ (associative law)

$\equiv p \vee T$ (Negation law)

$\equiv T$ (Domination law)

$\square$

b) $(p \wedge q) \to (p \to q)$

$\equiv (p \wedge q) \to (\neg p \vee q)$ (implication law)

$\equiv \neg(p \wedge q) \vee (\neg p \vee q)$ (implication law)

$\equiv (\neg p \vee \neg q) \vee (\neg p \vee q)$ (De Morgan's law)

$\equiv (\neg q \vee \neg p) \vee (\neg p \vee q)$ (Commutative law)

$\equiv \neg q \vee (\neg p \vee \neg p) \vee q$ (associative law)

$\equiv \neg q \vee \neg p \vee q$ (Idempotent law)

$\equiv (\neg q \vee q) \vee \neg p$ (commutative law)

$\equiv T \vee \neg p$ (Negation law)

$\equiv T$ $\square$ (Domination law).

c) $((p \to q) \wedge (q \to r)) \to (p \to r)$

$\equiv ((\neg p \vee q) \wedge (\neg q \vee r)) \to (\neg p \vee r)$ (Implication law)

$\equiv \neg((\neg p \vee q) \wedge (\neg q \vee r)) \vee (\neg p \vee r)$ (Implication law)

$\equiv (\neg(\neg p \vee q) \vee \neg(\neg q \vee r)) \vee (\neg p \vee r)$ (De Morgan's law)

$\equiv ((p \wedge \neg q) \vee (q \wedge \neg r)) \vee (\neg p \vee r)$ (De Morgan's law and double negation law)

$\equiv ((p \wedge \neg q) \vee \neg p) \vee ((q \wedge \neg r) \vee r)$ (Distributive law)

$\equiv \Big(\big(p \vee \neg p\big) \wedge (\neg q \vee \neg p)\Big) \vee \Big(\big(q \vee r\big) \wedge (\neg r \vee r)\Big)$ (Distributive law)

$\equiv \big(T \wedge (\neg q \vee \neg p)\big) \vee \big((q \vee r) \wedge T\big)$ (Negation law)

$\equiv (\neg q \vee \neg p) \vee (q \vee r)$ (Identity law)

$\equiv (\neg p \vee \neg q) \vee (q \vee r)$ (Commutative law)

$\equiv \neg p \vee (\neg q \vee q) \vee r$ (associative law)

$\equiv \neg p \vee T \vee r$ (Negation law)

$\equiv T$ (Domination law)

Ans 4 → $p \wedge q \vee r \equiv (p \wedge q) \vee r$

as $\wedge$ get higher precedence than $\vee$.

Ans 5 → Definitions from lecture notes.

ex → $p \vee \neg p$ is a tautology

$p \wedge \neg p$ is a contradiction

$p \vee q$ is a contingency.

Ans 6 → a) if $1+2=3$, i.e. true so

$x = x+1 \Rightarrow x = 2$

~~b) if $(x+1) = 2$~~

b) if $(x+1=3)$ or $(2x+2=3)$

put $x=1$, then

if $(1+1=3)$ or $(2\times1+2=3)$

false or false, i.e. false

so $x = x+1$ will not be executed

so answer $x=1$

c) if $(2\times1+3=5)$ and $(3\times1+4=7)$

if $(5=5)$ and $(7=7)$

true and true hence true

so $x = x+1$ will be executed $\Rightarrow x = 2$.

d) if $(1+1=2)$ XOR $(1+2=3)$

$(2=2)$ XOR $(3=3)$

True XOR True = False

so $x = x+1$ will not be executed

so answer $x=1$.

e) if $1<2$

it is false so answer $x=1$.