Tutorial Sheet - 4

Solution

Q-1.

a) p : I run

q : I will get there quicker.

The premises of argument are

P_1 : $P \Rightarrow q$

P_2 : q

& conclusion is Q : P

The Argument form is

$$\frac{\begin{array}{l} P_1 \\ P_2 \end{array}}{\therefore Q} \quad \text{or} \quad (P_1 \wedge P_2) \Rightarrow Q$$

which means

$$\frac{\begin{array}{l} P \Rightarrow q \\ q \end{array}}{\therefore P}$$

The Argument is invalid due to converse type of fallacy.

b) The atomic statements are same as in a).

Premises are P_1 : $P \Rightarrow q$

P_2 : $\neg P$

& Conclusion is Q : $\neg q$

The Argument given is

$$p \Rightarrow q$$
$$\neg p$$
$$\therefore \neg q$$

The given argument is invalid due to inverse type fallacy.

c) p : Bennett University is in Greater Noida.
q : Bennett University is in UP.

The premises are

$$p_1 : p \Rightarrow q$$
$$p_2 : q$$

& The conclusion Q : p

The given argument is

$$p \Rightarrow q$$
$$q$$
$$\therefore p$$

$\therefore$ Given argument is invalid.

d) p : Jatin is a policeman.
q : Jatin is a footballer.
r : Jatin has big feet.

The premises are $p_1 : p \vee q$
$p_2 : p \Rightarrow r$
$p_3 : \neg r$

and conclusion is Q : q

The given argument is

$$\begin{array}{l} p \vee q \\ p \Rightarrow r \\ \neg r \\ \hline \therefore \quad p \vee q \end{array}$$

$$\begin{array}{l} \neg p \quad (\because \text{By Modus Tollens with } P_2 \text{ \& } P_3) \\ \hline \therefore \quad q \quad (\because \text{By Disjunctive syllogism}) \end{array}$$

$\therefore$ The given argument is valid.

(But not sound as the 2nd premise is not true in real life)

c) p : The company invests in renewable energy.

q : The company will reduce its carbon footprint.

r : The company faces public backlash.

Here, premises are $P_1 : p \Rightarrow q$

$P_2 : p$

$P_3 : q \vee r$

& conclusion is Q : $\neg r$

The given argument is

$$\begin{array}{l} p \Rightarrow q \\ p \\ q \vee r \\ \hline \therefore \neg r \end{array}$$

By Modus Ponens for P_1 & P_2, we have

$$\frac{q \quad q \vee r}{\therefore \neg r}$$

∴ The argument is not valid due to the fallacy of the type disjunctive syllogism.

f) P : A student attend lectures of DMS.

q : student's understanding of CS will improve.

r : student is not present on campus.

s : DMS examination is not tough.

Here, Premises are $P_1 : P \Rightarrow q$

$P_2 : \neg q$

$P_3 : \neg P \Rightarrow r \vee s$

$P_4 : \neg r$

and the conclusion is Q : s

The argument is given as

$$\frac{\begin{array}{c} P \Rightarrow q \\ \neg q \\ \neg P \Rightarrow r \vee s \\ \neg r \end{array}}{\therefore \neg P \quad (\text{By Modus Tollens in } P_1 \text{ \& } P_2)}$$

$\neg P \Rightarrow r \vee s$

$\neg r$

$\therefore$ $r \vee s$ (By Modus ponens with $\neg p$ & p_3)

$\neg r$

$\therefore$ s (By Disjunctive Syllogism)

As the conclusion is infered from the given arguments, therefore argument is valid.

Q-2

a) $P(x)$: x is a lawyer.
$Q(x)$: x is a liar.

$\therefore$ $\forall x : P(x) \Rightarrow Q(x)$.

b) $\exists x : P(x) \Rightarrow \neg Q(x)$

c) $P(x)$: x is a person.
$Q(x)$: x becomes politician.
$R(x)$: x will get corrupt.

$\forall x : (P(x) \wedge Q(x)) \Rightarrow R(x)$.

d) $P(x)$: x is a doctor.
$Q(x)$: x know calculus.

$\forall x : P(x) \Rightarrow \neg Q(x)$.

e) $P(x,y)$: x is greter than y
& $Q(x)$: x is an integer

$$\forall x \exists y : (Q(x) \wedge Q(y) \wedge P(x,y)).$$

f) $P(x)$: x is an integer.
$Q(x)$: x is positive.
$R(x)$: x is negative.

$$\forall x : P(x) \Rightarrow (Q(x) \vee R(x))$$

Q-3

a) Let UoD = Universe of Indian festival
& $P(x)$: x is celebrated based on solar calender.
$Q(x)$: x is celebrated based on lunar calendar.

$\therefore$ $\forall x : P(x) \vee Q(x)$.

b) Let UoD = Universe of citizens.
$P(x)$: x is eligible to vote.
$Q(x)$: x is above 18 years of age.

$$\forall x : P(x) \Leftrightarrow Q(x).$$

c) Let UoD = Universe of family member.

$P(x)$: x enjoys spicy food.

$Q(x)$: x dislikes sweet

$\neg Q(x)$: x has a sweet tooth.

$R(x)$: x prefers mild flavors.

$$\forall x : (P(x) \wedge Q(x)) \vee (\neg Q(x) \wedge R(x))$$

d) $\forall x : (Even(x) \wedge x > 2) \Rightarrow (\exists p \exists q : Prime(p) \wedge Prime(q) \wedge (x = p + q))$

Q-4

a) No program invokes a function -
or
Program do not invoke any function.

b) There is only one book on the table.

c) All prime numbers greater than 2 are odd.

d) There is atleast one real number x for all real numbers such that addition of x in any real number result the same real number.
(Defining zero element).

Q-5

a) True → For $x = 9$, $R(9)$ is true

b) False → For $y = 10$

c) True → For any real number, there always exist one real number that is less than it.

d) False → There does not exist any real number which is greater or equal to all other real numbers.

e) True → For any two real numbers x & y, either $x > y$ or $x \leq y$.

f) True → As there exist at least one real number greater than 9 and all real numbers are not equal to 9.

g) False → There does not exist any real number greater than 9 which is greater that or equal to all real numbers.

h) True → As $\forall x : R(x)$ is false

$\Rightarrow \forall x \forall y : R(x) \wedge S(y)$ is false

$\Rightarrow \forall x \forall y : (R(x) \wedge S(y)) \Rightarrow Q(x, y)$ is true.