(1) (a) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

let $x \in \overline{A \cap B}$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in \overline{A}$ or $x \in \overline{B}$

$\Rightarrow x \in \overline{A} \cup \overline{B}$

$\Rightarrow \overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$ — (1)

now let $x \in \overline{A} \cup \overline{B}$

$\Rightarrow x \in \overline{A}$ or $x \in \overline{B}$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \in \overline{A \cap B}$

$\Rightarrow \overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$ — (2)

by (1) & (2), we can write

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

(b) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

let $x \in \overline{A \cup B}$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in \overline{A}$ and $x \in \overline{B}$

$\Rightarrow x \in \overline{A} \cap \overline{B}$

$\Rightarrow \overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$ — (1)

now let

$x \in \overline{A} \cup \overline{B}$

$\Rightarrow x \in \overline{A}$ and $x \in \overline{B}$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \in \overline{A \cup B}$

$\Rightarrow \bar{A} \cap \bar{B} \subseteq \overline{A \cup B}$ —— (2)

by (I) & (II) $\rightarrow$

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

Proved

(2) $A = \{1,2,3,4\}$, $B = \{1,3,5\}$

(a) $A \cup B = \{1,2,3,4,5\}$

(b) $A \cap B = \{1,3\}$

(c) $A - B = \{x \mid x \in A \text{ and } x \in B\} = \{2,4\}$

(d) $B - A = \{x \mid x \in B \text{ and } x \in A\} = \{5\}$

(e) $A \oplus B = (A-B) \cup (B-A) = \{2,4\} \cup \{5\} = \{2,4,5\}$

(3) $X = \{1,2,3,4,5,6,7\}$

The partition of a set X is the collection $P(X) = \{A_i \mid A_i \subseteq X\}$ satisfying the following properties:

i) all A_i's are non-empty.

ii) $A_i \cap A_j = \phi$, $\forall\, i,j$

iii) $\bigcup_i A_i = X$

So, only S_2 is partition.

· S_1 does not satisfy ii) condition.

· S_3 does not satisfy iii) condition.

(4) $X = \{10,20,30,40,50\}$

$\mu_A(x) = \dfrac{x}{x+10}$

$\mu_B(x) = 2x$

$\mu_B(x) = \frac{2x}{3x+10}$

So, $\mu_A(10) = \frac{10}{20} = 0.5$

$\mu_A(20) = \frac{20}{30} = 0.66$

$\mu_A(30) = \frac{30}{40} = 0.75$

$\mu_A(40) = \frac{40}{50} = 0.80$

$\mu_A(50) = \frac{50}{60} = 0.83$

$\mu_B(10) = \frac{20}{40} = 0.5$

$\mu_B(20) = \frac{40}{70} = 0.57$

$\mu_B(30) = \frac{60}{100} = 0.60$

$\mu_B(40) = \frac{80}{130} = 0.61$

$\mu_B(50) = \frac{100}{160} = 0.62$

a) So

$A = \{(10, 0.5), (20, 0.66), (30, 0.75), (40, 0.80), (50, 0.83)\}$

$B = \{(10, 0.5), (20, 0.57), (30, 0.60), (40, 0.61), (50, 0.62)\}$

b) and We know
c)

$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$

$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$

$\mu_{A^c}(x) = 1 - \mu_A(x)$

So, $A \cap B = \{(10, 0.5), (20, 0.57), (30, 0.60), (40, 0.61), (50, 0.62)\}$

$A \cup B = \{(10, 0.5), (20, 0.66), (30, 0.75), (40, 0.80), (50, 0.83)\}$

also we know,

$$|A| = \sum_{x \in X} \mu_A(x)$$

$|A \cap B| = 0.5 + 0.57 + 0.60 + 0.61 + 0.62 = 2.9$

$|A \cup B| = 0.5 + 0.66 + 0.75 + 0.80 + 0.83 = 3.54$

(5) $X = \{a, b, c, d\}$

$A = \{(a, 0.2), (b, 0.7), (c, 1), (d, 0.9)\}$

$B = \{(a, 0.1), (b, 0.8), (d, 0.2)\}$

$A \cap B = \{(a, 0.1), (b, 0.7), (d, 0.2)\}$

$|A \cap B| = 0.1 + 0.7 + 0.2 = 1$

$A \cup B = \{(a, 0.2), (b, 0.8), (c, 1), (d, 0.9)\}$

$|A \cup B| = 0.2 + 0.8 + 1 + 0.9 = 2.9$

$A^C = \{(a, 0.8), (b, 0.3), (c, 0), (d, 0.1)\}$

$|A^C| = 0.8 + 0.3 + 0 + 0.1 = 1.2$

(6) $|A| = 3$

No. of relation on $A = 2^{n^2}$

$= 2^{3^2} = 2^9 = 512$

(7) $A = \{1, 2, 3, 4\}$

$R = \{(a, b) / \text{ a divides b}\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

(8) A relation R on a set A is

Reflexive: If $(a,a) \in R, \forall a \in A$

Symmetric: If $(a,b) \in R$, then $(b,a) \in R, \forall a,b \in A$

Anti-symmetric: If $(a,b) \in R$ and $(b,a) \in R$, then $a = b, \forall a,b \in A$.

Transitive: If $(a,b) \in R$, $(b,c) \in R$, then $(a,c) \in R$. $\forall a,b,c \in A$.

So,

R_1 is not reflexive, not symmetric, not anti-symm., not transitive.

R_2 is not reflexive, not anti-symmetric, not transitive, but symmetric.

R_3 is reflexive, symmetric, but not transitive and anti-symmetric.

R_4 is not reflexive, not symmetric, but it is transitive and anti-symmetric.

R_5 is reflexive, transitive, anti-symmetric but not symmetric.

R_6 is not reflexive, not symmetric, but it is transitive and anti-symmetric.

(9) $n(D) = 36$, $n(A) = 12$, $n(S) = 18$,

$n(D \cup A \cup S) = 45$, $n(D \cap A \cap S) = 4$

No. of students who got two medals

$= n(A \cap D) + n(D \cap S) + n(A \cap S) - 2n(A \cap D \cap S)$

We know

$n(A \cup D \cup S) = n(A) + n(D) + n(S) - n(A \cap D) - n(A \cap S) - n(D \cap S) + n(A \cap D \cap S)$

$45 = 36 + 12 + 18 - n(A \cap D) - n(A \cap S) - n(D \cap S) + 4$

$\Rightarrow n(A \cap D) + n(A \cap S) + n(D \cap S) = 25$

So required answer $= 25 - 2 \times 4 = 17$

(10) Required answer is

$n(A \cap D) + n(D \cap S) + n(A \cap S) - 3n(A \cap D \cap S)$

$= 25 - 3 \times 4 = 13$

= No. of students who got exactly two

models.