



Course Name: **Discrete Mathematical Structure**

Course Code: **CSET106**

Academic Year: 2023-24

Semester: Even

Date: 5 April 2024

Type: 3-1-0

Tutorial Sheet: **9**

Batch: 2023–27

CO-Mapping

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
CO1									
CO2	✓	✓	✓	✓	✓	✓	✓	✓	✓
CO3			✓				✓		

Objectives

1. Students will be able to apply Chinese Remainder Theorem.
2. Students will be able to differentiate between a permutation and a combination. Besides, they will be able to evaluate number of permutations and combinations.
3. Students will be able to understand the applications related to congruence and inverse congruence.
4. Students will be able to apply the idea of recurrence relations and generating functions.

Questions

1. Apply Chinese Remainder Theorem (CRT) to solve the following system of congruence equations:
 - (a) $x \equiv 5 \pmod{7}$ and $x \equiv 4 \pmod{9}$.
 - (b) $x \equiv 1 \pmod{2}$, $x \equiv 2 \pmod{3}$ and $x \equiv 3 \pmod{7}$.
2. The child of a number theorist is sorting a large pile of coins (worth less than Rs. 50) into groups of 3 coins each. At the end, the child reports that 2 coins are left over. The child

starts over, instead sorting the coins into groups of 4 and reports that 1 coin is left over. The child starts over again, sorting the coins into groups of 5 and reports that 1 coins are left over. The number theorist didn't originally know how many pennies were in the pile, but at this point he speaks up the correct number. Does the number theorist have enough information to tell how many coins are in the pile? What is that number?

Hint: $x \equiv 2 \pmod{3}$, $x \equiv 1 \pmod{4}$, $x \equiv 1 \pmod{5}$

3. Find the number of ways of considering 3 students out of 10 students.
4. Find the number of ways to create a committee consisting of a president, a secretary and a treasurer of 3 students out of 10 students.
5. How many ways 10 students be arranged: (a) in a row, (b) around a circular table?
6. Find the number of ways to arrange 10 students in a row if 3 students always come together.
7. How many ways 3 alphabets can be picked from 10 distinct alphabets
 - (a) if repeataion of alphabets are allowed?
 - (b) if repeataion of alphabets are not allowed?
8. Find the coefficient of x^{50} in the expansion of $(1 + x)^{100}$.
9. Derive the generating function for the Fibonacci sequence.

$$F_n = F_{n-1} + F_{n-2} \quad \forall n \geq 2, F_0 = 1 = F_1$$

10. Consider the recurrence relation:

$$a_n = 2a_{n-1} + 3a_{n-2} + 2^{n-1} \quad \forall n \geq 2$$

and with initial conditions $a_0 = 1, a_1 = 2$. Evaluate the value of $3a_3 + 2a_2 - a_1$.

“Strive for perfection in everything you do. Take the best that exists and make it better. When it does not exist, design it.” - Sir Henry Royce