Department of Mathematics
School of Computer Science Engineering and Technology
Bennett University

Course Code: EMAT102L | **Course Name: Linear Algebra & ODEs**
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	CO1	CO2	CO3	CO4	CO5
Q1		✓			
Q2		✓			
Q3		✓			
Q4		✓			
Q5		✓			
Q6		✓			

Tutorial Sheet 3

1) Prove that the set $U = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}; a, b \in \mathbb{R} \right\}$, is a subspace of the vector space $M_{2\times2}(\mathbb{R})$ of 2×2 matrices.

2) Which of the following sets of vectors $x = (x_1, x_2, x_3)^t$ in $\mathbb{R}^3$ are subspaces of $\mathbb{R}^3$?

   (a) All x such that $x_1 \geq 0$,
   (b) All x such that $x_1 + 3x_2 = x_3$,
   (c) All x such that $x_2 = x_1^2$,
   (d) All x such that $x_1 x_2 = 0$,
   (e) All x such that x_2 is rational,
   (f) All x such that $x_1^2 + x_2^2 + x_3^2 \leq 1$.

3) Express $v = \begin{bmatrix} 2 & -5 & 3 \end{bmatrix}^t$ in a linear combination of $u_1 = \begin{bmatrix} 1 & -3 & 2 \end{bmatrix}^t, u_2 = \begin{bmatrix} 2 & -4 & 1 \end{bmatrix}^t$, and $u_3 = \begin{bmatrix} 1 & -5 & 7 \end{bmatrix}^t$.

4) Express $M = \begin{bmatrix} 4 & 7 \\ 7 & 9 \end{bmatrix}$ as a linear combination of A, B, C where $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$.

5) Show that the vectors $v_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^t, v_2 = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^t, v_3 = \begin{bmatrix} 1 & 5 & 8 \end{bmatrix}^t$ span $\mathbb{R}^3$.

6) Determine whether or not the matrices v_1 and v_2 are linearly dependent where $v_1 = \begin{bmatrix} 1 & 3 & -4 \\ 5 & 0 & -1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -4 & -12 & 16 \\ -20 & 0 & 4 \end{bmatrix}$.

7) Determine whether the set $\{[1,2,0]^t, [0,1,-1]^t, [1,1,2]^t\}$ is linearly independent in $\mathbb{R}^3$.

8) Show that the vectors $v_1 = \begin{bmatrix} 1+i & 2i \end{bmatrix}^t$ and $v_2 = \begin{bmatrix} 1 & 1+i \end{bmatrix}^t$, in $\mathbb{C}^2$ are linearly dependent over the field $\mathbb{C}$ but linearly independent over the field $\mathbb{R}$.

9) Determine whether or not each of the following forms a basis of $\mathbb{R}^3$:

(a) $v_1 = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^t, v_2 = \begin{bmatrix} 1 & 3 & 5 \end{bmatrix}^t, v_3 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^t, v_4 = \begin{bmatrix} 2 & 3 & 0 \end{bmatrix}^t$.

(b) $v_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^t, v_2 = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^t, v_3 = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix}^t$.

(c) $v_1 = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}^t, v_2 = \begin{bmatrix} 1 & 2 & 5 \end{bmatrix}^t, v_3 = \begin{bmatrix} 5 & 3 & 4 \end{bmatrix}^t$.

10) (a) Show that the set $\{x^2+1, 3x-1, -4x+1\}$ is linearly independent in $\mathbb{P}_2(\mathbb{R})$.

(b) Show that the set $\{x+1, x-1, -x+5\}$ is linearly dependent in $\mathbb{P}_1(\mathbb{R})$.

11) What is the span of $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$?

12) Prove that the set $V = \left\{ \begin{bmatrix} a & b & c \\ 0 & d & f \\ 0 & 0 & g \end{bmatrix} \in \mathcal{M}_{3\times 3}(\mathbb{R}) : a+b+c=0, a+d+g=0 \right\}$ is a vector space of $\mathcal{M}_{3\times 3}(\mathbb{R})$ and find a basis for it and its dimension.