Department of Mathematics
School of Computer Science Engineering and Technology
Bennett University

Course Name:	**Linear algebra & ODE**	Course Code:	**EMAT102L**
Academic Year:	2023-24	Semester:	Even
Date:	16/02/2024	Type:	Core (L-T-P: 3-1-0)
Tutorial Sheet:	**5**		

CO-mapping:

	CO1	CO2	CO3	CO4	CO5	CO6
Q1		✓				
Q2		✓				
Q3		✓				
Q4		✓				
Q5		✓				
Q6		✓				

Objectives: Students will be able to understand Linear Transformation: (Examples, Range space, Null space/Kernel, Nullity, Rank, Matrix representation of linear transformations, etc.).

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1. Verify whether the mapping T is a linear transformation:

   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + y, x - y)$.

   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x + y, x - 2y + 1)$.

   (c) $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (x + y, x - y + z)$.

   (d) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (xy, yz, zx.)$

   (e) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x + y, y + z, x + 2y + z)$.

(f) $T : M_2(\mathbb{R}) \to \mathbb{R}^3$ defined by $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a+b, a-b, a)$.

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(1,1,0) = (1,2,3)$ and $T(0,1,1) = (1,0,1)$. Then find the value of $T(2,3,1)$ and $T(2,0,0)$. Utilizing the Rank-Nullity Theorem, explore the feasible dimensions for the null space of the linear transformation T.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(1,1) = (1,2)$ and $T(0,1) = (2,1)$. Then find the value of $T(2,3)$. Is it possible to find the linear transformation value at any point of $\mathbb{R}^2$? If so, find $T(x,y)$.

4. In question number 1., if T is a linear transformation, then find $\text{Ker}(T)$ (i.e. $N(T)$) and $\text{Range}(T)$ (i.e. $R(T)$). Also find rank and nullity of those linear transformations.

5. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator given by

$$T(x,y,z) = (3x+y, x+z, x-z).$$

Find the matrix of T with respect to the standard basis of $\mathbb{R}^3$.

6. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear operator given by

$$T(x,y,z) = (-2x+3z, x+2y-z).$$

Find the matrix for T with respect to the ordered bases $B_1 = \{(1,1,1), (1,1,-1), (1,-1,1)\}$ for $\mathbb{R}^3$ and $B_2 = \{(2,1), (1,2)\}$ for $\mathbb{R}^2$.

"The only way to learn Mathematics is to do Mathematics."– Paul R. Halmos