Department of Mathematics
School of Computer Science Engineering and Technology
Bennett University

Course Code: EMAT102L **Course Name: Linear Algebra & ODEs**
Academic Year: 2023-24 **Semester: Even**
Date: 14/04/2024 **Type: Core (L-T-P: 3-1-0)**

	CO1	CO2	CO3	CO4	CO5
Q1					✓
Q2					✓
Q3					✓
Q4					✓
Q5					✓

Tutorial Sheet 10

1) Consider the linear differential equation

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0.$$

(a) Show that x^3 and $|x^3|$ are two linearly independent solutions of the differential equation on $x \in (-\infty, \infty)$.

(b) x^3 and $|x^3|$ are two linearly independent solutions of the differential equation but $W(x^3, |x^3|) = 0, \forall x \in \mathbb{R}$. Does it violate any result? Explain.

(c) x^2 and x^3 are also two linearly independent solutions of the differential equation. Can we write general solution of the differential equation in terms of these solutions?

2) Use reduction of order method to find the second linearly independent solution of the following differential equations

(a) $(x-1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0; \quad y_1(x) = x.$

(b) $x^2\frac{d^2y}{dx^2} - (2a-1)x\frac{dy}{dx} + a^2y = 0; a \neq 0, x > 0, \quad y_1(x) = x^a.$

(c) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = 0; \quad y_1(x) = x\sin(logx).$

Also write the general solution for each differential equation.
Solutions: (a) $y_2 = e^x$. (b) $y_2 = x^a logx$. (c) $y_2 = -x\cos(logx)$.

3) Find the second order differential equation corresponding to given linearly independent solutions

(a) $y_1 = \cos 2\pi x,\ y_2 = \sin 2\pi x.$

(b) $y_1 = e^{-\sqrt{2}x},\ y_2 = xe^{-\sqrt{2}x}.$

(c) $y_1 = e^{(-1+i\sqrt{2})x},\ y_2 = e^{(-1-i\sqrt{2})x}.$

Solutions: (a) $y'' + 4\pi^2 y = 0$. (b) $y'' + 2\sqrt{2}y' + 2y = 0$. (c) $y'' + 2y' + 3y = 0$.

4) Solve the IVP's

(a) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0; \quad y(-1) = e,\ y'(-1) = -\frac{e}{4}.$

(b) $\frac{d^2y}{dx^2} - k^2 y = 0;\ k \neq 0, \quad y(0) = 1,\ y'(0) = 1.$

(c) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0; \quad y(0) = 3,\ y'(0) = -1.$

(d) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0; \quad y(0) = 2,\ y'(0) = -3.$

(e) $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0; \quad y(0) = 1,\ y'(0) = -8,\ y''(0) = -4.$

5) Solve the following non-homogeneous differential equation

(a) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \frac{e^{-3x}}{x^3}.$

Solutions: $y = \left(c_1 + c_2 x + \frac{1}{2x}\right) e^{-3x}.$