Tutorial Sheet 2 Solutions

1) Let there be x number of Ƶ 1 bills,
y number of Ƶ 5 bills, and
z number of Ƶ 10 bills.

Then we have
$$\begin{cases} x+y+z = 32 & \text{—— (1)} \\ x+5y+10z = 100 & \text{—— (2)} \end{cases}.$$
and x, y, z all nonnegative integers.

(2) $-$ (1) gives

$$4y+9z = 68$$

$$\Rightarrow \quad z = \frac{68-4y}{9}$$

$$= \frac{72-4(y+1)}{9}$$

$$= 8 - \frac{4}{9}(y+1)$$

our target is to make z a +ve integer.
So if we choose $\boxed{y=8}$, then z becomes an integer.

Choose $\boxed{y=8}$

$$z = 8-4$$
$$\Rightarrow \boxed{z=4}$$

$$x+8+4 = 32$$
$$\Rightarrow \boxed{x=20}$$

$\therefore$ There are <u>20</u> Ƶ 1, <u>8</u> Ƶ 5, and <u>4</u> Ƶ 10 notes present.

2) Let there be

x number of tickets for adults

y number of tickets for children.

Then we have

$$\begin{cases} x + y = 74 & \text{—①} \\ 325x + 175y = 19100 & \text{—②} \end{cases}$$

two eq^ns in two variables.

② − 175×① gives

$$150x = 19100 - 74\times 175 = 19100 - 12950 = 6150$$

$$\boxed{x = 41}$$

$$\Rightarrow \boxed{y = 33}$$

3) $a = 3$.

4)
$$\begin{cases} x+y+z=2 & \text{—①} \\ x+2y+3z=1 & \text{—②} \\ y+2z=0 & \text{—③} \end{cases}$$

③ − (② − ①) gives.

$$y+2z - ((x+2y+3z)-(x+y+z)) = 0 - (1-2)$$

$$\Rightarrow 0 = 1. \quad \Rightarrow\!\!\Leftarrow.$$

In eq^n ③

$$y+2z = \boxed{0}$$

if 0 will be replaced by −1, then we get infinitely many sol^ns.

In this case we'll have the following system

$x + y + z = 2$ note that ... we get the 3rd eq^n.

$$\begin{cases} [illegible] \\ x + 2y + 3z = 1 \\ y + 2z = -1 \end{cases}$$ Substracting [illegible]

$$\Rightarrow \boxed{y = -1 - 2z}.$$

So if $\boxed{z = t} \in \mathbb{R}$ (a parameter)

then $\boxed{y = -1 - 2t}$

and
$$\begin{aligned} x &= -y - z + 2 \\ &= -(-1-2t) - t + 2 \\ &= 1 + 2t - t + 2 \end{aligned}$$

$$\Rightarrow \boxed{x = 3 + t}$$

$\therefore$ There are infinitely many sol^{n}s to the system and the sol^{n}s are

$$x = 3 + t, \; y = -1 - 2t, \; z = t \quad \forall t \in \mathbb{R}.$$

For picking one sol^n, one can choose $t = 1$.

So $\boxed{x = 4, \; y = -3, \; z = 1}$

3) a) $\begin{cases} x + y + z = 10 \\ x + y = 2 \\ z = 1 \end{cases}$

b) $\begin{cases} x + y + z = 10 \\ y + z = 5 \\ z = 4 \end{cases}$

c) $\begin{cases} x + y + z = 10 \\ x + y = 2 \\ z = 8 \end{cases}$

[illegible]

no sol^n | exactly one sol^n | infinitely many sol^{n}s.

4) a) $\begin{cases} 2x_1 + 5x_2 = 1 \\ x_1 + 4x_2 = 2 \end{cases}$

$\Delta = \begin{vmatrix} 2 & 5 \\ 1 & 4 \end{vmatrix} = 8-5 = 3$

$\Delta_{x_1} = \begin{vmatrix} 1 & 5 \\ 2 & 4 \end{vmatrix} = 4-10 = -6$

$\therefore$ By Cramer's rule

$\Delta_{x_2} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4-1 = 3$

$x = \frac{\Delta_{x_1}}{\Delta} = \frac{-6}{3} \Rightarrow \boxed{x_1 = -2}$

$y = \frac{\Delta_{x_2}}{\Delta} = \frac{3}{3} \Rightarrow \boxed{x_2 = 1}$

b) $x = 3,\ y = -1,\ z = -2.$

Hint compute $\Delta, \Delta_x, \Delta_y, \Delta_z$.

7) Try to make two consecutive zeros (e.g. using row operations, reduce (2,1) and (3,1) entries to zeros, then the determinant's order will be reduced.

8) Let A be a skew symmetric matrix of order $= 2k+1$ for some $k \in \mathbb{N}$.

$A^T = -A$

$\Rightarrow \det(A) = \det(A^T) = \det(-A) = (-1)^{2k+1} \det(A)$

$$= -\det(A)$$

$$\Rightarrow 2\det(A) = 0 \quad \Rightarrow \det(A) = 0 .$$

3. Ans =3, -3. Recall that solution is unique if and only if rank(A)=ran([A|b])= numbers of variable.

To find the values of a for which there is a whole line of solutions, we can examine the determinant of the coefficient matrix. The system of equations can be written in matrix form as:

$$\begin{bmatrix} a & 3 \\ 3 & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The determinant of the coefficient matrix is given by $\det = a^2 - 9$.

For the system to have a unique solution (x = y = 0), the determinant must be non-zero. So, we set $a^2 - 9 \neq 0$ and solve for a:

$$a^2 - 9 \neq 0$$

$$(a + 3)(a - 3) \neq 0$$

This equation is satisfied for all values of a except $a = -3$ and $a = 3$. Therefore, for all values of a except $a = -3$ and $a = 3$, the system of equations has a unique solution, and for these values, there is a whole line of solutions.