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Batch 73.

Tutorial-7

1) a) $x^2 dy + y^2 dx = 0$

$\Rightarrow x^2 dy = -y^2 dx$

$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{x^2}$

so, the equ^n is non-linear as the dependent variable degree is more than 1.

order - 1

Degree - 1

b) $\frac{d^2y}{dx^2} + x\sin y = 0$

The equ^n is non-linear as the dependent variable is inside a trig. function.

Degree - Not defined (not in polynomial form)

order - 2

c) $\frac{d^2y}{dx^2} + \frac{d^4y}{dx^4}\frac{d^3y}{dx^3} + y = x$ linear as degree is 1.

order - 4

Degree - 1

d) $\left(\frac{dy}{dx}\right)^3 = \sqrt{\frac{d^2y}{dx^2} + 1}$

$\left(\frac{dy}{dx}\right)^6 = \frac{d^2y}{dx^2} + 1$

Non-linear as degree of diff. coeff is 6.

order - 2 Degree - 1

2) a) $\frac{dy}{dx} = y - y^2$, $y = \frac{1}{1+Ce^{-x}}$, $y(0) = \frac{1}{4}$

To verify that y is a sol^n of the, o.

let's solve. LHS = RHS

so, $\frac{dy}{dx} = y - y^2$

$$\frac{d\left(\frac{1}{1+ce^{-x}}\right)}{dx} = \frac{1}{1+ce^{-x}} = \frac{1}{(1+ce^{-x})^2}$$

$$\frac{-1}{(1+ce^{-x})^2} \times (1+ce^{-x})' = \frac{1+ce^{-x}-1}{(1+ce^{-x})^2}$$

$$\frac{-1}{(1+ce^{-x})^2} \times (-ce^{-x}) = \frac{ce^{-x}}{(1+ce^{-x})^2}$$

so, $\frac{ce^{-x}}{(1+ce^{-x})} = \frac{ce^{-x}}{(1+ce^{-x})^2}$

$\therefore$ LHS = RHS

Hence proved

$y(0) = \frac{1}{4}$

$y = \frac{1}{1+ce^{-x}} = \frac{1}{4}$

$\frac{1}{1+c} = \frac{1}{4}$

$1+c = 4$

$c = 3$

so, $\boxed{y = \frac{1}{1+3e^{-x}}}$

b) $\frac{dy}{dx} = y + e^x$, $y = (x+c)e^x$, $y(0) = \frac{1}{2}$

so, $\frac{d(x+c)e^x}{dx} = (x+c)e^x + e^x$

$(x+c)'e^x + (x+c)e^x = [(x+c)+1]e^x$

$e^x + (x+c)e^x = (x+c)e^x + e^x$

$\therefore$ LHS = RHS

Hence, verified.

Now, $\because y(0) = \frac{1}{2}$

$\frac{1}{2} = (0+c)e^0$ 3.

$\Rightarrow \boxed{c = \frac{1}{2}}$

$\Rightarrow$ so, $\boxed{y = \left(x + \frac{1}{2}\right)e^x}$ //

3) a) $\frac{dy}{dx} = (x+1)e^{-x}y^2$

$\Rightarrow \frac{dy}{dx} = (x+1)e^{-x}dx = (xe^{-x} + e^{-x})dx$

$\Rightarrow \frac{-1}{y} = (-x-1)e^{-x} - e^{-x} + C$

$\Rightarrow -\frac{1}{y} = -xe^{-x} - e^{-x} - e^{-x} + C$

$\Rightarrow \boxed{\frac{y}{g} = \frac{1}{(x+2)e^{-x} - C}}$ //

b) $\frac{dy}{dx} = \sec^2 y$

$\Rightarrow \frac{dy}{\sec^2 y} = dx$

$\Rightarrow \int \cos^2 y \, dy = \int dx$

$\Rightarrow \frac{1}{2}\left(y + \frac{1}{2}\sin 2y\right) + C = x$

$\Rightarrow \boxed{x = \frac{y}{2} + \frac{\sin 2y}{4} + C}$ //

c) $2xy\frac{dy}{dx} = y^2 - x^2$

$\Rightarrow 2xy\frac{dy}{dx} = y^2 - x^2$

$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{2x} - \frac{x}{2y}$ —(1)

let $y = vx$, $v = y/x$

$\Rightarrow$ so, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

so from (1), $\frac{dy}{dx} = \frac{v}{2} - \frac{1}{2v}$

$\Rightarrow \frac{dy}{dx} = \frac{v^2-1}{2v}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{v^2-1}{2v}$

$\Rightarrow dv = \frac{dx}{x}\left(\frac{v^2-1-2v^2}{2v}\right)$

$\Rightarrow \frac{dv}{\cdot} = \frac{dx}{x}\left(\frac{-v^2-1}{2v}\right)$

$\Rightarrow \frac{2v}{-v^2-1}\,dv = \frac{dx}{x}$ $\left\{ \begin{array}{l} \text{let } v^2+1 = t \\ 2v\,dv = dt \end{array} \right\}$

$\Rightarrow$ so, $\frac{dt}{-t} = \frac{dx}{x}$

Integrate both sides,

$\Rightarrow -\ln tc = \ln x$ (where, c = const.)

$\Rightarrow -\ln(v^2+1)c = \ln x$

$\Rightarrow \ln\left(\frac{1}{v^2+1}\right)c = \ln x$

$\Rightarrow \ln\left(\frac{x^2}{y^2+x^2}\right)c = \ln x$

$\Rightarrow \frac{cx^2}{x^2+y^2} = x$

$\Rightarrow \boxed{cx = x^2+y^2}$

d) $x\frac{dy}{dx} = y + 3x^4\cos^2 x\,(\cancel{y}/x)$ $y(1) = 0$

$\Rightarrow x\frac{dy}{dx} = y + 3x^4\cos^2 x\,(y/x)$ $y(1) = 0$

if $y = vx$, $y/x = v$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow$ so, $x\left(v + x\frac{dv}{dx}\right) = vx + 3x^4\cos^2 v$

$\Rightarrow vx + x^2\frac{dv}{dx} = vx + 3x^4\cos^2 v$

$\Rightarrow x^2\frac{dv}{dx} = 3x^4\cos^2 v$

$\Rightarrow \int \frac{dv}{\cos^2 v} = \int 3x^2 dx$

$\Rightarrow \int \sec^2 v \, dv = \frac{3x^3}{3}$, $\sec^2 v = 1 + \tan^2 v$

$\Rightarrow \tan(v) + C = x^3$

$\Rightarrow C + \tan\frac{y}{x} = x^3 - C$

$\Rightarrow y = x\tan^{-1}(x^3 - C)$

$\Rightarrow \because y(1) = 0$

$\Rightarrow$ so, $0 = 1 \cdot \tan^{-1}(1 - C)$

$\Rightarrow 0 = \tan^{-1}(1 - C)$

$\Rightarrow \tan 0 = 1 - C$

$\Rightarrow \therefore \boxed{C = 1}$

so, $\boxed{y = x\tan^{-1}(x^3 - 1)}$

4) a) $(x\tan(y/x) + y)\,dx - x\,dy = 0$

$\Rightarrow \left(x\tan\frac{y}{x} + y\right)dx = x\,dy$

$\Rightarrow (x\tan v + vx)\,dx = x\,dy$

$\Rightarrow (x\tan v + vx) = x\left(v + x\frac{dv}{dx}\right)$

$\Rightarrow x\tan v + vx = vx + x^2\frac{dv}{dx}$

$\Rightarrow \int \frac{dv}{\tan v} = \int \frac{dx}{x}$

$\Rightarrow \int \frac{\cos v}{\sin v} dv = \ln x + \ln c$, let $\sin v = t$

$\cos v = \frac{dt}{dv}$

$\cos v \, dv = dt$

$\Rightarrow \int \frac{dt}{t} = \ln x + \ln c$

$\Rightarrow \ln \sin v = \ln x + \ln c$

$\Rightarrow \sin v = cx$

$\Rightarrow \sin(y/x) = cx$

$\Rightarrow \boxed{y = x\sin^{-1}(cx)}$

6) $(5x+2y+1)dx + (2x+y+1)dy = 0$

$\Rightarrow \frac{-(5x+2y+1)}{2x+y+1} = \frac{dy}{dx}$

$\Rightarrow$ let $x = x_1+h$, $y = y_1+k$,

$\Rightarrow \frac{dy}{dx} = \frac{-5(x_1+h)-2(y_1+k)-1}{2(x_1+h)+(y_1+k)+1}$

$\Rightarrow \frac{dy}{dx} = \frac{-5x_1-5h-2y_1-2k-1}{2x_1+2h+y_1+k+1}$

$\Rightarrow \frac{(-5x_1-2y_1)-(5h+2k+1)}{(2x_1+y_1)+(2h+k+1)}$

$\Rightarrow -5h-2k-1 = 0$ — (I)

$2h+k+1 = 0$ — (II)

$\Rightarrow$ on solving, (I) & (II), we get,

$\Rightarrow h = 1, \; k = -3$

$\Rightarrow$ so, $\frac{dy}{dx} = \frac{-5x_1-2y_1}{2x_1+y_1}$

$\Rightarrow 2x_1dy_1 + y_1dy_1 = -5x_1dx_1 - 2y_1dx_1$

$\Rightarrow 2(x_1dy_1 + y_1dx_1) + 5x_1dx_1 + y_1dy_1 = 0$

$d(x_1 y_1) + 5x_1\,dx_1 + y_1\,dy_1 = 0$ 7.

Integrating each term,

$\Rightarrow 2x_1y_1 + \frac{5x_1^2}{2} + \frac{y_1^2}{2} = C$

$\Rightarrow 4x_1y_1 + 5x_1^2 + y_1^2 = C$

$\Rightarrow \cancel{4(x_1+h)(y_1+k) + 5(x_1+h)^2 + (y_1+k)^2 = C}$

$\Rightarrow \cancel{4(x_1y_1 + x_1k + hy_1 + hk) + 5(x_1^2 + h^2 + 2x_1h) + (y_1^2 + k^2 + 2y_1k) = C}$

$\Rightarrow \cancel{4x_1y_1 + 4x_1k + 4hy_1 + 4hk + 5x_1^2 + 5h^2 + 10x_1h + y_1^2 + k^2 + 2y_1k = C}$

$\Rightarrow \cancel{4x_1y_1 + 12x_1 + 4y_1 + 12 + 5x_1^2 + 10x_1 + 5 + y_1^2 + 9 + 6y_1 = C}$

$\Rightarrow \cancel{\boxed{5x_1^2 + 4x_1y_1 + y_1^2 + 12x_1 + 10y_1 = C}}$

$\cancel{+12}$

$\Rightarrow 4(x-h)(y-k) + 5(x-h)^2 + (y-k)^2 = C$

$\Rightarrow 4(x-1)(y+3) + 5(x-1)^2 + (y+3)^2 = C$

$\Rightarrow 4(xy + 3x - y - 3) + 5(x^2 + 1 - 2x) + (y^2 + 9 + 6y) = C$

$\Rightarrow 4xy + 12x - 4y - 12 + 5x^2 + 5 - 10x + y^2 + 9 + 6y = C$

$\Rightarrow 4xy + 5x^2 + y^2 + 2x + 2y + 2 = C$

$\Rightarrow \boxed{4xy + 5x^2 + y^2 + 2x + 2y = C}$