Department of Mathematics
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	CO1	CO2	CO3	CO4	CO5
Q1	✓				
Q2	✓				
Q3	✓				
Q4	✓				
Q5	✓				
Q6	✓				

Tutorial Sheet 1

1. What are the possible reduced row echelon form of each of a 2×2 and a 3×3 matrix?

2. Find the row echelon form of each of the following matrices. Further, reduce them into reduced row echelon Form.

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 5 & 6 & 2 \\ -1 & 2 & 4 & 3 \\ 1 & 2 & -1 & 2 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \text{ and } \begin{bmatrix} 3 & 4 & 5 & -6 \\ 2 & 3 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 5 & -5 & 5 & 5 \end{bmatrix}$$

3. Solve the following two systems of linear equations, both of them have the same matrix of coefficients. $x_1 - x_2 + 3x_3 = b_1,\ 2x_1 - x_2 + 4x_3 = b_2,\ -x_1 + 2x_2 - 4x_3 = b_3$ for $[b_1, b_2, b_3]^t = [0, 1, 2]^t, [3, 3, -4]^t$.

4. Solve the following systems of equations using Gaussian elimination method as well as Gauss-Jordan elimination method, whenever they are consistent:

   (a) $x + y + z = 3,\ x - y - z = -1,\ 4x + 4y + z = 9;$

   (b) $-x + y + z + w = 0,\ x - y + z + w = 0,\ -x + y + 3z + 3w = 0,\ x - y + 5z + 5w = 0;$

   (c) $x + y + 2z = 3,\ -x - 3y + 4z = 2,\ -x - 5y + 10z = 11;$

   (d) $2w + 3x - y + 4z = 0,\ 3w - x + z = 1,\ 3w - 4x + y - z = 2.$

5. For what values of $\lambda \in \mathbb{R}$ and $k \in \mathbb{R}$, the following systems of equations has (i) no solution, (ii) a unique solution, and (iii) infinitely many solutions?

   (a) $x + y + z = 3,\ x + 2y + \lambda z = 4,\ 2x + 3y + 2\lambda z = k;$ and

   (b) $x + y + 2z = 3,\ x + 2y + \lambda z = 5,\ x + 2y + 4z = k.$

Also, find the solutions whenever they exist.

6. Let A be an $n \times n$ matrix. If the system $A^2x = 0$ has a non-trivial solution then show that the system $Ax = 0$ also has a non-trivial solution.