Department of Mathematics
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Course Code: EMAT102L **Course Name: Linear Algebra & ODEs**
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	CO1	**CO2**	**CO3**	**CO4**	**CO5**
Q1			✓		
Q2			✓		
Q3			✓		
Q4			✓		
Q5			✓		
Q6			✓		
Q7			✓		
Q8			✓		

Tutorial Sheet 6

1. If $\lambda = 0$ is an eigenvalue of A, then show that A is singular.

2. Prove that

   (a) Similar matrices have the same characteristic polynomial.

   (b) Similar matrices have the same eigenvalues.

3. Let $A = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{bmatrix}$. Then find

   (a) The characteristic polynomial of A and the corresponding eigenvectors.

   (b) The corresponding eigenvectors.

4. Prove that the eigenvalues of a triangular matrix are the entries on its main diagonal.

5. Show that the following matrices A, B and C are diagonalizable. Also, find invertible matrices S_1, S_2 and S_3 such that $S_1^{-1}AS_1, S_2^{-1}BS_2$ and $S_3^{-1}CS_3$ are all diagonal matrices.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 3 & 2 & -2 \\ 0 & 3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}, \quad \text{and} \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

6. Let A be an $n \times n$ matrix with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. Show that $\det(A) = \lambda_1 \cdot \lambda_2 \cdots \lambda_n$ and $\text{trace}(A) = \lambda_1 + \lambda_2 + \cdots + \lambda_n$.

7. Let A be 2×2 matrix such that $\text{trace}(A) = 4$ and $\det(A) = 5$. Find the eigenvalues of A.

8. A 3×3 matrix A has characteristic polynomial $\lambda(\lambda - 1)(\lambda + 2)$. What is the characteristic polynomial of A^2?